

Scattergraph Principles and Practice

Practical Application of the Froude Number to Flow Monitoring

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ABSTRACT

Engineers are aware of the instability associated with critical flow conditions and are generally advised to avoid them during sewer design. However, such conditions are often encountered in existing sewers and can impact the accuracy and reliability of flow monitoring data.

The Froude number (Fr) is a dimensionless number used to describe flow conditions within a sewer. These conditions can be illustrated on a scattergraph using iso-Froude lines. Certain flow conditions such as hydraulic jumps, sewer bores, and undular jumps are readily identified by evaluating flow monitoring data with respect to iso-Froude lines.

The concept of the iso-Froude is presented and developed in this paper. Practical examples from flow monitoring locations throughout the United States are also provided, demonstrating the scattergraph signatures of various transcritical and near-critical flow phenomena. Flow monitors can operate well in sewers under subcritical or supercritical conditions, but accuracy may deteriorate near the transition. Such conditions should be avoided when possible in flow monitoring applications.

KEY WORDS

Flow Monitoring, Froude Number, iso-Froude, Scattergraph

Introduction

Engineers are aware of the instability associated with critical flow conditions and are generally advised to avoid them during sewer design.¹ However, such conditions are often encountered in existing sewers and can impact the accuracy and reliability of flow monitoring data.

The scattergraph is a graphical tool that provides insight into sewer performance through a simple and intuitive display of flow monitoring data.² Through the application of the Froude number, the scattergraph can also be used to identify transcritical and near-critical flow conditions that may impact flow monitoring results. This application is developed and discussed in this paper.

Iso-Froude Lines

The Froude number (Fr) is a dimensionless number used to describe flow conditions in a sewer and is defined in Equation (1).

$$Fr = \frac{v}{\sqrt{gd_h}} \quad (1)$$

where: Fr = Froude number
 v = flow velocity, ft/s
 g = gravitational acceleration, ft/s²
 d_h = hydraulic mean depth, ft

The hydraulic mean depth (d_h) introduced in Equation (1) is defined in Equation (2).

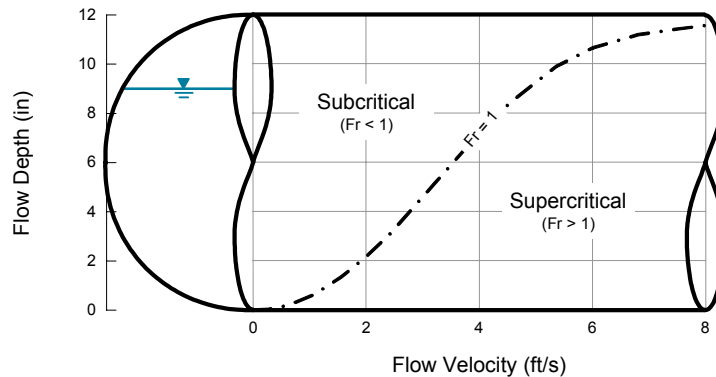
$$d_h = \frac{A}{B} \quad (2)$$

where: d_h = hydraulic mean depth, ft
 A = wetted area, ft²
 B = surface width, ft

Flow conditions within a sewer are directly related to the Froude number. If $Fr < 1$, flow conditions are classified as *subcritical* and are often described as tranquil or streaming. If $Fr > 1$, flow conditions are classified as *supercritical* and are often described as rapid or shooting. If $Fr = 1$, flow conditions are classified as *critical* and are often described as unstable.¹

These conditions are illustrated on a scattergraph using an iso-Froude line as shown in Figure 1. For this example, an iso-Froude line is constructed for $Fr = 1$. Subcritical conditions occur to the left of the iso-Froude line, and supercritical conditions occur to the right.¹

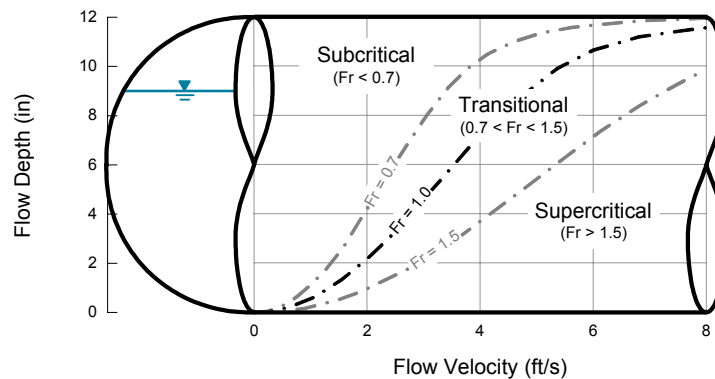
FIGURE 1: Scattergraph with Iso-Froude Line for $Fr = 1$



Based on theoretical considerations and experimental observations, Hager and others have modified and expanded the traditional classification of flow conditions.³ The most important modification is the recognition of a region of *transitional flow* ($0.7 < Fr < 1.5$) between subcritical and supercritical flow where unstable conditions may be observed.

These conditions are illustrated on a scattergraph by constructing iso-Froude lines at the upper and lower boundaries of the transitional region as shown in Figure 2. Subcritical conditions occur to the left of the lower boundary, transitional conditions occur between the boundaries, and supercritical conditions occur to the right of the upper boundary.

FIGURE 2: Scattergraph with Multiple Iso-Froude Lines



According to Hager, these boundaries are “somewhat arbitrary” but are sufficient to provide a general description of local flow conditions.³ These conditions can be identified with *human viewing speed* by evaluating flow monitoring data with respect to iso-Froude lines.

Constructing Iso-Froude Lines

Constructing iso-Froude lines requires a simple algebraic rearrangement to express Equation (1) in terms of the flow velocity associated with a specified Froude number as a function of flow depth (d). This equation is shown in Equation (3):

$$v = Fr \sqrt{gd_h} \tag{3}$$

- where:
- v = flow velocity, ft/s
 - Fr = Froude number
 - g = gravitational acceleration, ft/s²
 - d_h = hydraulic mean depth, ft

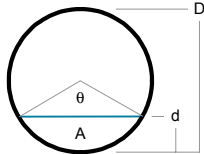
An iso-Froude is constructed by solving Equation (3) for $0 < d < D$ and plotting the results on a scattergraph. An example is provided to demonstrate the concept.

EXAMPLE Construct an iso-Froude line ($Fr = 1$) on a scattergraph for a 24-in sewer.

Solution

Use Equation (3) to calculate v for $d = 4, 8, 12, 16, 20,$ and 24 in.

For a circular sewer,¹



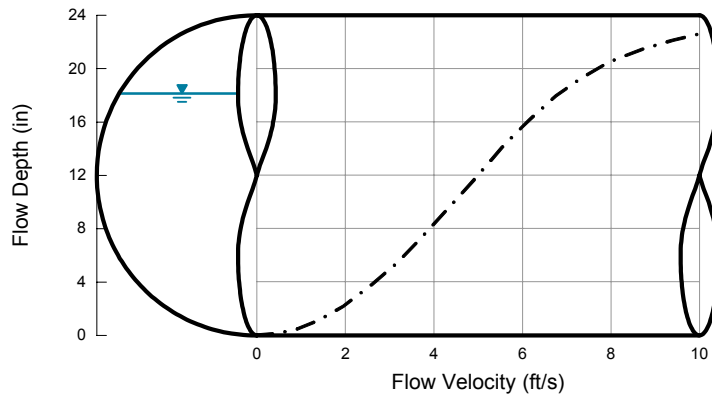
| d | θ | A | B | d_h | v |
|-----|----------|-----------------|------|-------|------|
| in | ° | ft ² | ft | ft | ft/s |
| 0 | 0 | 0.00 | 0.00 | | |
| 4 | 96 | 0.34 | 1.49 | 0.23 | 2.73 |
| 8 | 141 | 0.92 | 1.89 | 0.49 | 3.96 |
| 12 | 180 | 1.57 | 2.00 | 0.79 | 5.03 |
| 16 | 219 | 2.22 | 1.89 | 1.18 | 6.16 |
| 20 | 264 | 2.80 | 1.49 | 1.88 | 7.77 |
| 24 | 360 | 3.14 | 0.00 | | |

$$\theta = 2\cos^{-1}(1 - 2d/D)$$

$$A = (D^2/8)(\theta - \sin \theta)$$

$$B = D \sin (\theta/2)$$

These results provide the necessary information to construct the iso-Froude line on a scattergraph, as shown below:



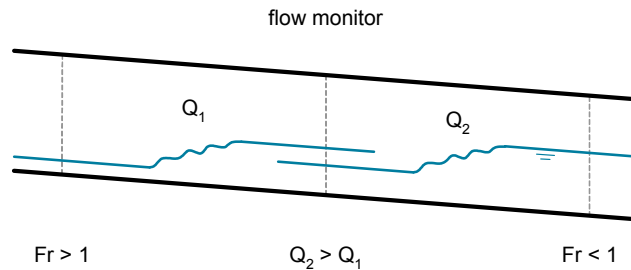
Transitional Conditions

Certain flow conditions such as hydraulic jumps, sewer bores, and undular jumps exhibit distinct signatures on a scattergraph and are readily identified by evaluating flow monitoring data with respect to iso-Froude lines.⁴ Examples of each are provided in the following sections, along with a discussion of the impact of transitional conditions on flow monitoring applications.

Hydraulic Jump

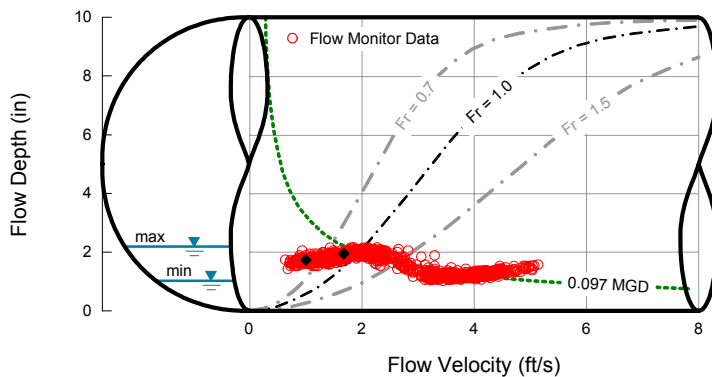
A *hydraulic jump* occurs when flow transitions from supercritical to subcritical flow. Supercritical conditions are observed on the upstream side of the jump, and subcritical conditions are observed on the downstream side of the jump, as shown in Figure 3.

FIGURE 3: Hydraulic Jump



A hydraulic jump may result from a variety of downstream conditions, including a transition from a steep to mild slope, an obstruction, or other related conditions. A scattergraph depicting a hydraulic jump is shown in Figure 4.

FIGURE 4: Scattergraph of a Hydraulic Jump

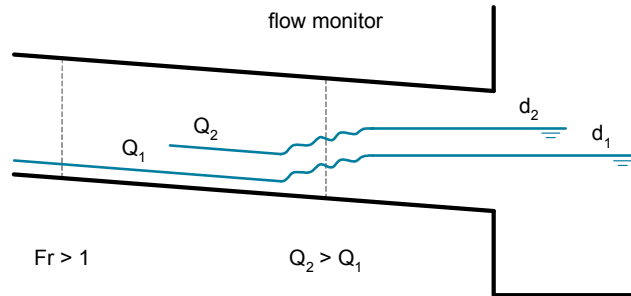


In this example, the hydraulic jump is located upstream from the monitor at lower flow rates (Q_1), and the subcritical side of the hydraulic jump is observed. As the flow rate increases (Q_2), the hydraulic jump is *pushed* through the monitoring location, and the supercritical side of the hydraulic jump is observed. This scattergraph shows a distinct and repeatable shift in the data pattern that occurs at a specific flow rate. An iso- Q^{TM} line is used to denote the flow rate at which this condition occurs.⁵

Sewer Bore

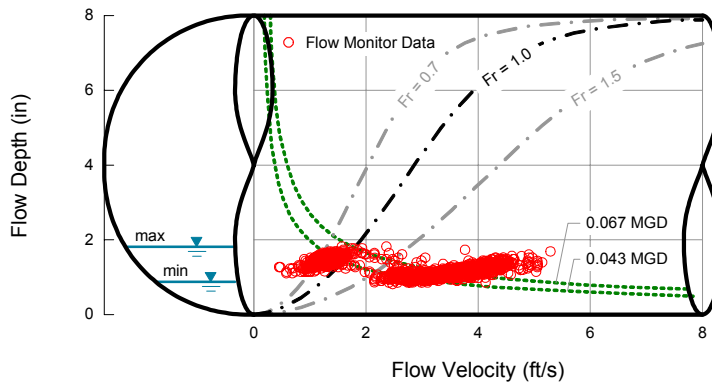
A *sewer bore* is similar to a *tidal bore* in which a rising tide will cause a hydraulic jump to migrate upstream on a river or channel.⁴ This condition can form in an incoming sewer if the flow depth is influenced by a larger downstream sewer or a pump station wet well, as shown in Figure 5.

FIGURE 5: Sewer Bore



As the downstream water level rises, a hydraulic jump migrates upstream as a sewer bore. A scattergraph believed to depict a sewer bore is shown in Figure 6.

FIGURE 6: Scattergraph of a Sewer Bore

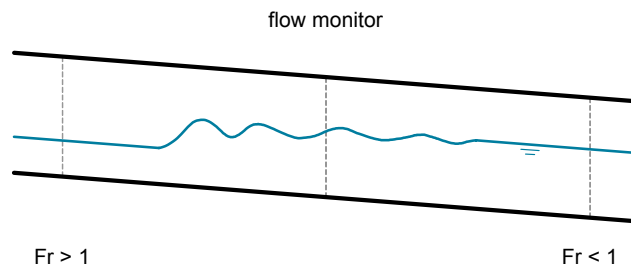


The key difference between a traditional hydraulic jump and a sewer bore is the nature of the downstream conditions. A fixed downstream condition, such as a transition from a steep to mild slope, will result in a traditional hydraulic jump that occurs repeatedly at the same flow depth and same flow rate at the monitoring location. A variable downstream condition, such as the flow depth in an intercepting sewer, will result in a hydraulic jump that can occur within a variable range of flow depths and flow rates as the sewer bore passes the monitoring location. This scattergraph shows two distinct and repeatable shifts in the data pattern that occur at two different flow rates. Iso-Q lines are used to denote the flow rates at which the sewer bore was observed to pass through the monitoring location.

Undular Jump

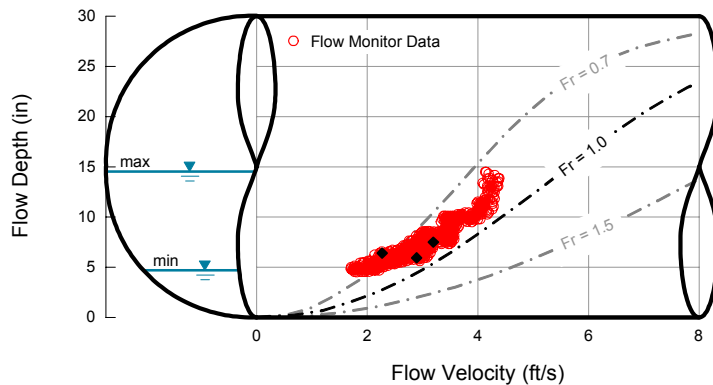
Near-critical flows are defined as those that exhibit critical or near-critical flow conditions over an extended distance and include undular jumps, surges, and oscillating flows.⁶ An undular jump is depicted in Figure 7.

FIGURE 7: Undular Jump



This condition is different from a traditional hydraulic jump and includes surface undulations and disturbances that extend some distance downstream from the jump.⁶ A scattergraph believed to depict near-critical flow conditions resulting from an undular jump is shown in Figure 8.

FIGURE 8: Scattergraph of an Undular Jump



This scattergraph is the result of the surface undulations on the downstream side of the undular jump and is characterized by a *stair step* pattern. The maximum distance between wave crest and trough is nearly five inches and occurs when the beginning of the undular jump reaches its closest approach to the flow monitor.

Discussion

Flow monitors can operate well in either subcritical or supercritical conditions, but accuracy may deteriorate during the transition. Field testing of flow monitors sponsored by the Environmental Protection Agency (EPA) under the Environmental Technology Verification (ETV) program has been previously reported and demonstrates one example of this effect.⁷ Two flow monitors were installed within the same sewer, along with a variety of reference measurement devices, at a test location in the City of Sainte-Foy, Quebec, Canada in 2001. During dry weather flow simulations, both flow monitors reported similar results, with average reported flow rates within 5% of reference measurements. However, during wet weather simulations, the reported flow rates between the two flow monitors deviated by up to 25%. Field observations revealed that a stationary wave appeared at one of the monitoring locations during the wet weather simulation, in this case overstating the actual flow rate.

This example raises concerns regarding how often transitional flow conditions are encountered in routine flow monitoring applications. To investigate this concern, flow monitoring data were surveyed from 255 flow monitoring locations around the United States. Flow depth and velocity data were plotted on a scattergraph and evaluated with respect to iso-Froude lines. The results of this survey indicate that 56% of the monitoring locations are described as subcritical flow, 36% of the monitoring locations are described as transitional flow, and 8% of the monitoring locations are described as supercritical flow. Additional research is warranted to further classify the transitional flow conditions that might be encountered, as well as their effect on the accuracy of flow depth and velocity measurements and corresponding flow rate calculations.

Conclusion

Engineers are aware of the instability associated with critical flow conditions and are generally advised to avoid them during sewer design. The same advice applies to flow monitoring in existing sewer systems where deviations in reported flow rates on the order of 25% have been documented. Transitional flow conditions – including hydraulic jumps, sewer bores, and undular jumps – can be identified on a scattergraph by evaluating flow monitoring data with respect to iso-Froude lines. This technique can be used to determine if transitional flow conditions are present at a monitoring location and assess if such conditions might impact the accuracy and reliability of flow monitoring data.

Symbols and Notation

The following symbols and notation are used in this paper:

VARIABLES

| | |
|----------------|---|
| d | = flow depth, ft |
| v | = flow velocity, ft/s |
| Q | = flow quantity, ft ³ /s |
| g | = gravitational acceleration, ft/s ² |
| d _h | = hydraulic mean depth, ft |
| A | = wetted area, ft ² |
| B | = surface width, ft |
| D | = diameter, ft |
| Fr | = Froude number |

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References

1. Butler, D. and Davies, J.W. (2000). *Urban Drainage*. E & FN Spon, London.
2. Enfinger, K.L. and Keefe, P.N. (2004). "Scattergraph Principles and Practice – Building a Better View of Flow Monitoring Data," KY-TN Water Environment Association Water Professionals Conference; Nashville, TN.
3. Hager, W.H. (1999). *Wastewater Hydraulics – Theory and Practice*. Springer, Berlin.
4. Stevens, P.L., Kimbrough, H.R, and Enfinger, K.L. (2004). "Be Wary of Weally Wough Waves," *Proceedings of the Collection Systems Specialty Conference*; Milwaukee, WI; Water Environment Federation: Alexandria, VA.
5. Enfinger, K.L. and Stevens, P.L. (2006). "Scattergraph Principles and Practice – Tools and Techniques to Evaluate Sewer Capacity," *Proceedings of the Pipeline Division Specialty Conference*; Chicago, IL; American Society of Civil Engineers: Reston, VA.
6. Chanson, H. (1995). "Flow Characteristics of Undular Hydraulic Jumps – Comparison With Near-Critical Flows," *Research Report CH45/95*. Department of Civil Engineering, University of Queensland, Australia.
7. NSF International (2003). "ADS Environmental Model 4000 Open Channel Flow Monitor," *Environmental Technology Verification Report*; EPA/600/R-04/034; Washington, D.C.